Plasma flows near a reconnecting current layer: strong magnetic field approximation

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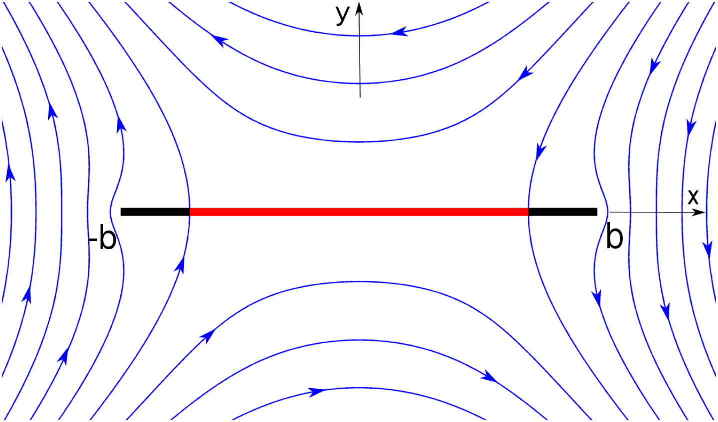
- *2D* velocity field $\mathbf{v}(x, y, t)$.
- ***Strong*** magnetic field $\mathbf{B}(x, y, t)$.
- The density $\rho(x, y, t)$.



The ideal MHD system of partial differential equations

$$\frac{d\mathbf{v}}{dt} = -\frac{\nabla p}{\rho} - \frac{1}{4\pi\rho}[\mathbf{B} \times rot\mathbf{B}]$$

$$\frac{\partial \mathbf{B}}{\partial t} = rot[\mathbf{v} \times \mathbf{B}]$$

$$\frac{\partial \rho}{\partial t} + div\rho\mathbf{v} = 0$$

The dimensionless equations and the approximation used

$$\frac{\varepsilon^2}{\delta}\frac{\partial \mathbf{v}}{\partial t} + \varepsilon^2(\mathbf{v}\nabla)\mathbf{v} = -\gamma^2\frac{\nabla p}{\rho} - \frac{1}{\rho}[\mathbf{B} \times rot\mathbf{B}],$$

$$\frac{\partial \mathbf{B}}{\partial t} = \delta rot[\mathbf{v} \times \mathbf{B}], \quad \frac{\partial \rho}{\partial t} + \delta div\, \rho\mathbf{v} = 0,$$

where

$$\delta = \frac{VT}{L} = 1, \quad \varepsilon^2 = \frac{v^2}{V_A^2}, \quad \gamma^2 = \frac{p_0^2}{\rho_0 V_A^2}.$$

Strong field and cold plasma $\gamma^2 << \varepsilon^2 << 1.$

The equations in the zeroth-order in ε^2

$$\begin{cases} rot\mathbf{B} = \mathbf{0}, & (1) \\ \mathbf{B}\dfrac{d\mathbf{v}}{dt} = 0, & (2) \\ \dfrac{\partial \mathbf{B}}{\partial t} = rot[\mathbf{v} \times \mathbf{B}], & (3) \\ \dfrac{\partial \rho}{\partial t} + div\,\rho\mathbf{v} = 0. & (4) \end{cases}$$

2D approximation ([1])

We consider 2D magnetic fields and plasma motions:

$$\mathbf{v} = \{v_x(x, y, t), v_y(x, y, t), 0\},$$

$$\mathbf{B} = \{B_x(x, y, t), B_y(x, y, t), 0\},$$

$$\mathbf{j} = \{0, 0, j(x, y, t)\},$$

$$\mathbf{A} = \{0, 0, A(x, y, t)\}.$$

[1] Somov B.V., Plasma Astrophysics, Part I, Fundamentals and Practice, Second Edition, Springer SBM, 2012, New York

2D approximation

$$F = A(x,y,t) + iA^{+}(x,y,t),$$

$$A^{+}(x,y,t) = \int \left(-\frac{\partial A}{\partial y}dx + \frac{\partial A}{\partial x}dy\right) + A^{+}(t),$$

$$z = x + iy,$$

$$\mathbf{B} = B_x + iB_y = -i\overline{\frac{dF}{dz}}.$$

Set of the equations in terms of $A(x, y, t)$

$$\begin{cases} (1) \\ (2) \\ (3) \\ (4) \end{cases} \Longrightarrow \begin{cases} \triangle A = 0, & (5) \\ [\frac{d\mathbf{v}}{dt} \times \nabla A] = \mathbf{0}, & (6) \\ \frac{dA}{dt} = 0, & (7) \\ \frac{\partial \rho}{\partial t} + div\, \rho \mathbf{v} = 0. & (8) \end{cases}$$

Plasma kinematics

According to the previous system the plasma kinematics is described by the following ordinary differential equations **for x and y**

$$\begin{cases} \ddot{x}\dfrac{\partial A}{\partial y} - \ddot{y}\dfrac{\partial A}{\partial x} = 0, & \textbf{(6)} \\ \\ \dot{x}\dfrac{\partial A}{\partial x} + \dot{y}\dfrac{\partial A}{\partial y} + \dfrac{\partial A}{\partial t} = 0. & \textbf{(7)} \end{cases}$$

The Syrovatskii current layer

The complex potential of the Syrovatskii CL is given by the following formula ([2])

$$F(z,t) = \frac{\alpha}{2} z\sqrt{z^2 - b^2} + \frac{\Gamma}{2\pi} Ln\frac{z + \sqrt{z^2 - b^2}}{b} + A(t)$$

where

$$\Gamma = -\pi\alpha b^2 \nu \qquad \nu \in [0,1]$$

is the coefficient proportional to the full current in the layer.

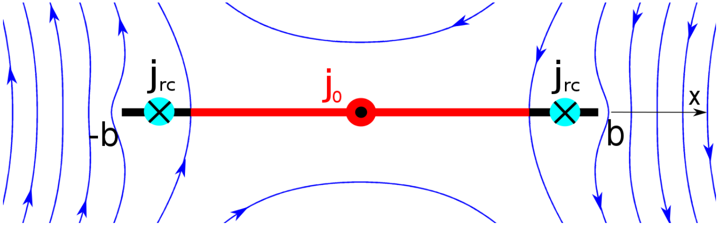
[2]Syrovatskii, Soviet Physics JETP, 1971, Volume 33, № 5

Stationary current layer

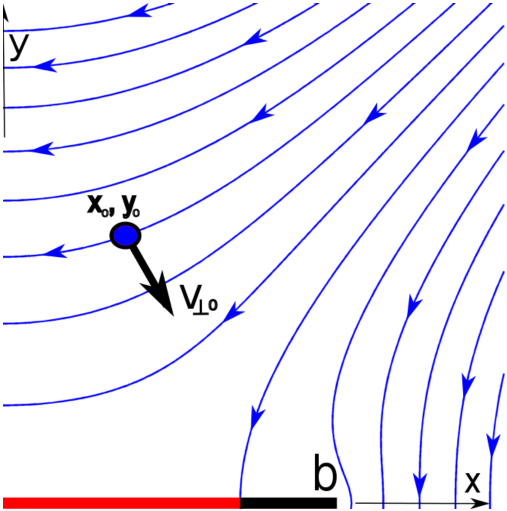
The CL with permanent width of $2b$

$$A(x,y,t) = A(x,y) + A(t), \quad A(t) = A(0) - t,$$

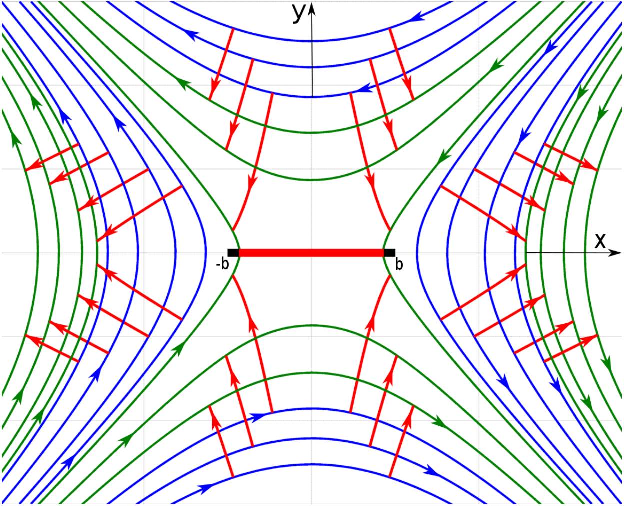
$$A(0) = \frac{b^2}{4} - \nu \frac{b^2}{2} ln \frac{b}{2l}.$$



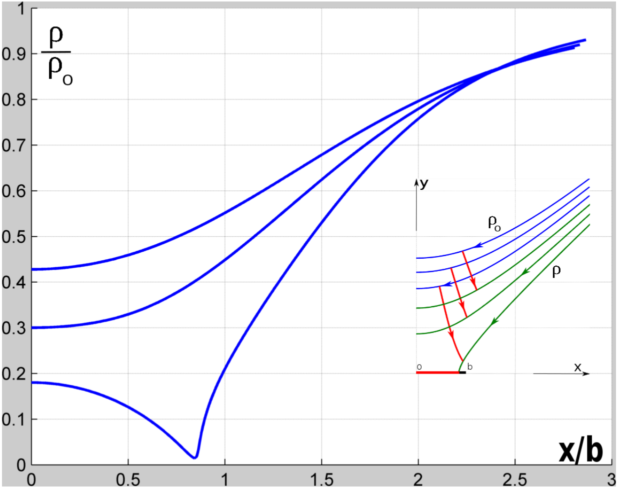
Initial conditions



Numerical results: flow pattern

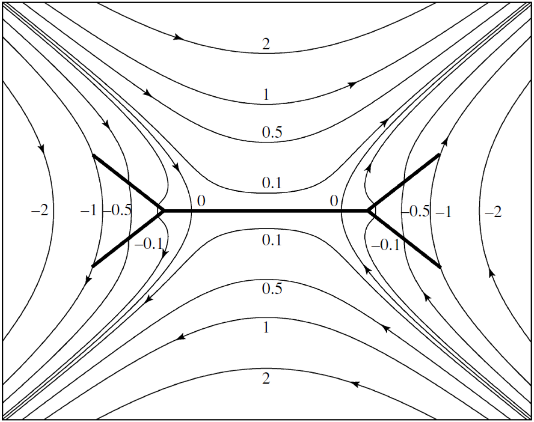


Density distribution along the field lines



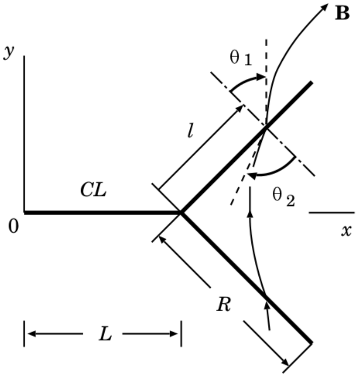
The RCL with attached MHD shocks

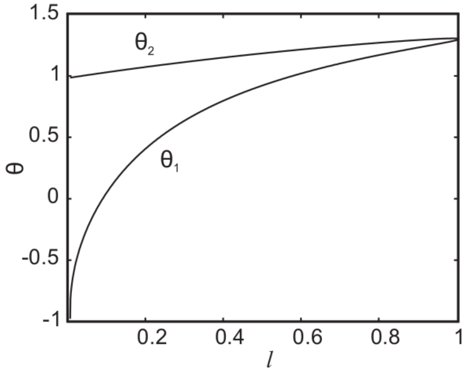
The method allows to calculate the velocity field in a more general case:



[3] Bezrodnykh, Vlasov, Somov, Astronomy Letters, **37** (2), 113-130, 2011.

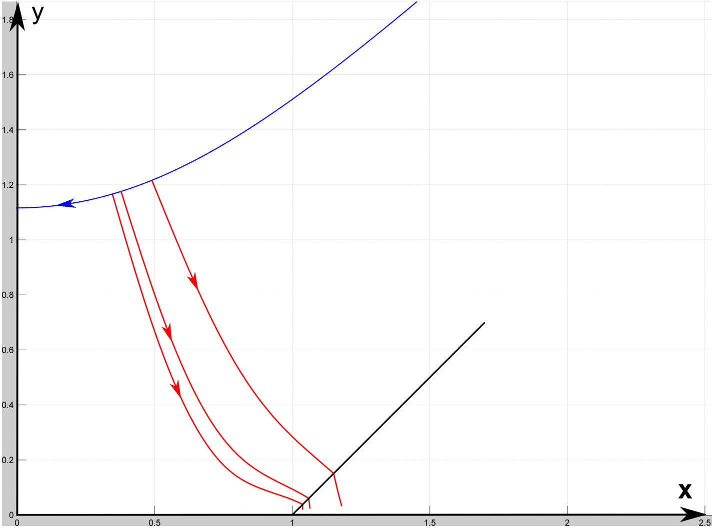
The fast shock wave



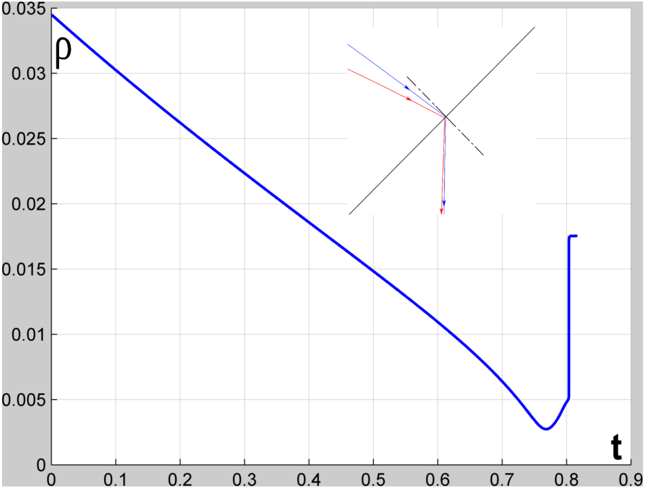


[4] Ledentsov, L.S. and Somov, B.V.: Evolutionary of Discontinuous Plasma Flows in the Vicinity of Reconnecting Current Layers. Astrophys Space Sci Proc. 30, 117–131 (2012)

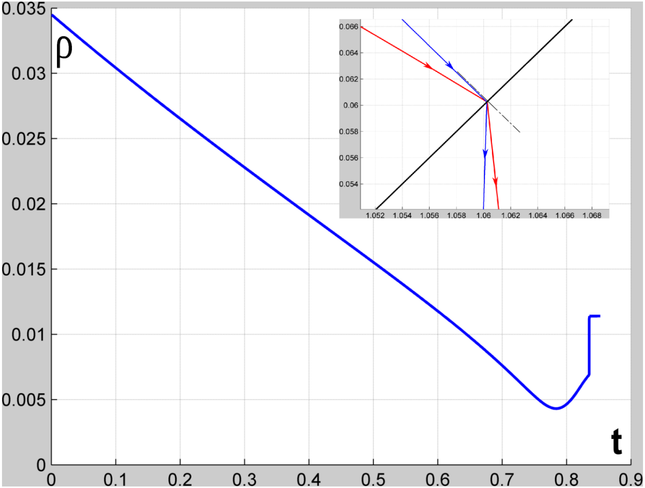
Flow pattern near the shock wave



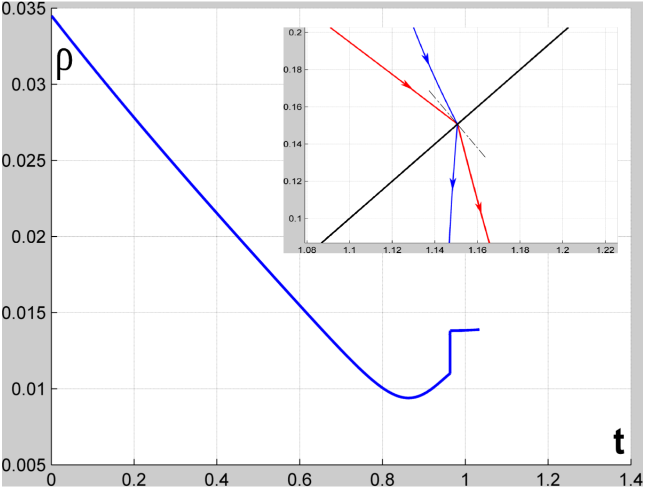
Plasma density profile along the track *TA*



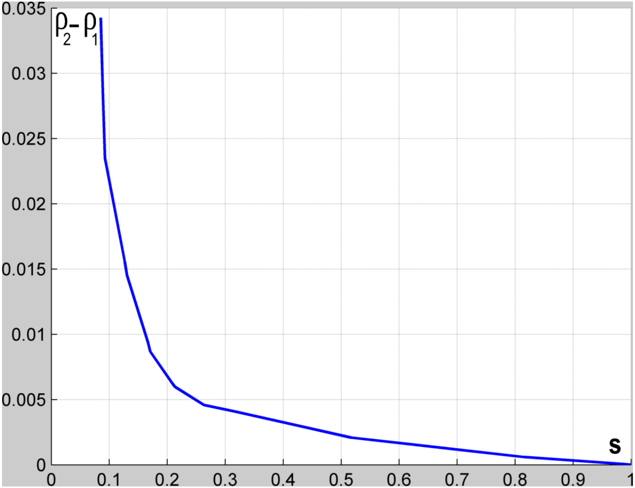
Plasma density profile along the track S_{on}



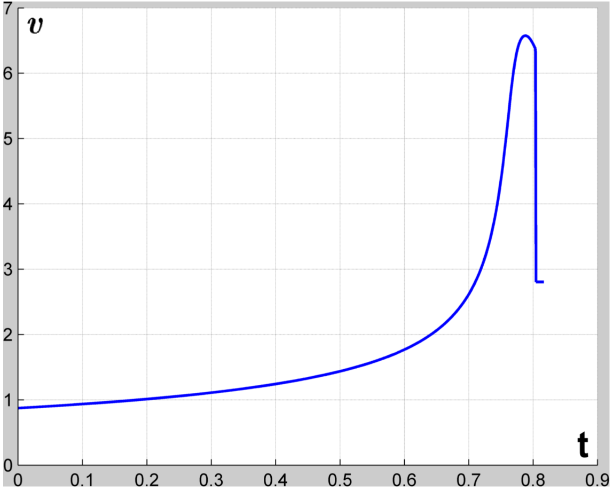
Plasma density profile along the track S_+



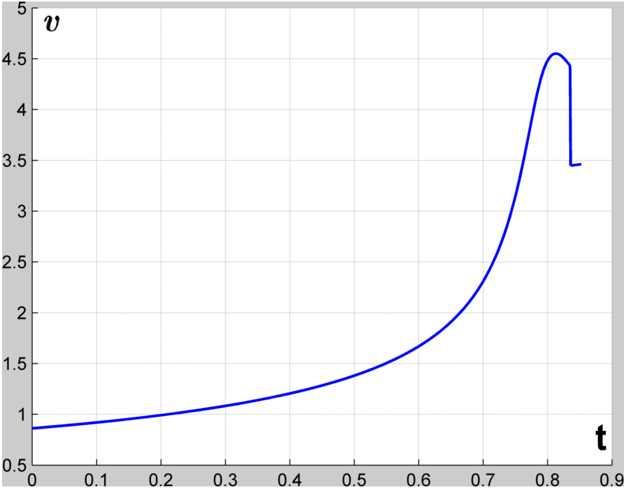
Density jump along the shock wave



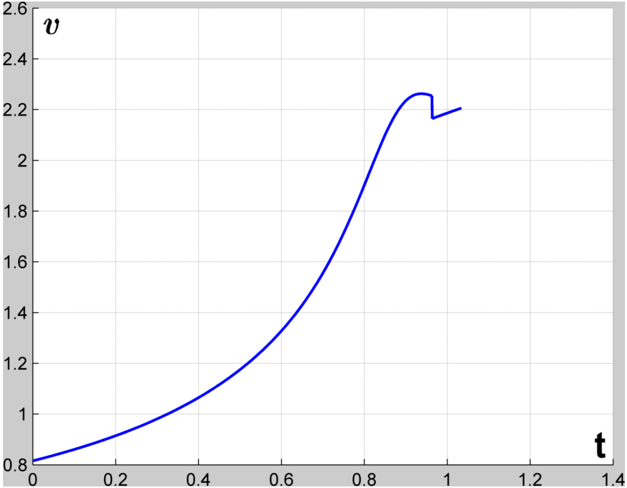
Plasma velocity profile along the track TA



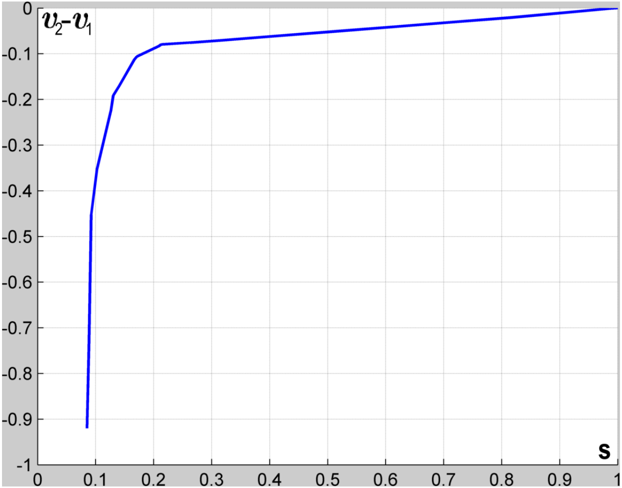
Plasma velocity profile along the track S_{on}



Plasma velocity profile along the track S_+



Velocity jump along the shock wave



Conclusions

- The method allows to solve a system of ordinary differential equations instead of partial differential equations system.

- The method used is applicable for the investigation of the generalized model of a reconnecting current layer with attached MHD discontinuities.